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Moreover the remaining factors will give you a *positive* product. For, if one of the factors be

$$[(\xi_1+i\eta_1)-(\xi_2+i\eta)]^2=[(\xi_1-\xi_2)+i(\eta_1-\eta_2)]^2$$

there will be a corresponding factor

$$[(\xi_1-i\eta_1)-(\xi_2-i\eta_2)]^2=[(\xi_1-\xi_2)-\mathrm{i}(\eta_1-\eta_2)]^2.$$

When multiplied these give

$$[(\xi_1-\xi_2)^2+(\eta_1-\eta_2)^2]^2,$$

which is clearly positive. So all the factors containing complex roots may be taken in pairs which give a positive product, and, of course, the factors containing real roots only are positive. Hence corresponding to every pair of complex roots there will be *one* and *only one* negative factor. Therefore, if the number of pairs be even the discriminant will be positive; if odd, negative.

NOTE.=Mr. Harry S. Vandiver should have been credited as a joint author of the solution of problem 102.

GEOMETRY.

129. Proposed by **WILLIAM HOOVER, A. M., Ph. D.,** Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Show that at no point of an ellipse will the circle of curvature pass through the center, if the eccentricity be less than $\frac{1}{2}\sqrt{2}$.

I. Solution by **F. ANDEREGG, A.M.,** Professor of Mathematics, Oberlin College, Oberlin, O.; **W.H. CARTER, A. M.,** Professor of Mathematics, Centenary College, Jackson, La.

Since for the ellipse the radius of curvature is

$$\rho=\frac{(a^4y^2+b^4x^2)^{\frac{3}{2}}}{a^4b^4};$$

and the center of curvature is the point

$$\left(\frac{(a^2-b^2)x^3}{a^4},\ -\frac{(a^2-b^2)y^3}{b^4}\right),$$

the equation of the circle of curvature is

$$\left(x'-\frac{(a^2-b^2)x^3}{a^4}\right)^2+\left(y'+\frac{(a^2-b^2)y^3}{b^4}\right)^2=\frac{(a^4y^2+b^4x^2)^3}{a^8b^8}.$$

This circle passes through the origin

$$(a^2-b^2)^2(b^8x^6+a^8y^6)=(a^4y^2+b^4x^2)^3.$$

This equation is easily simplified, and assumes the form

$$\frac{x^2}{a^2}=\frac{1+e^2}{3e^2}.$$

It follows at once that the least value of $3e^2$ is $1+e^2$, or the least value of e is $\frac{1}{2}\sqrt{2}$.

II. Solution by the PROPOSER.

If ϕ be the eccentric angle of any point of the ellipse $a^2y^2+b^2x^2=a^2b^2$, the equation to the corresponding circle of curvature is

$$x^2+y^2-(a^2-b^2)\left(\frac{2\cos^3\phi}{a}x-\frac{2\sin^3\varphi}{b}y\right)$$

$$+a^2(\cos^2\varphi-2\sin^2\varphi)-b^2(2\cos^2\varphi-\sin^2\varphi)=0.$$

This passing through the center, requires that

$$a^2(\cos^2\phi-2\sin^2\varphi)-b^2(2\cos^2\varphi-\sin^2\varphi)=0,$$

$$\text{or }\frac{a^2-b^2}{a^2}=e^2=\frac{1}{2-3\sin^2\varphi},$$

which is a minimum for $\phi=0$ or π; that is $e^2=\frac{1}{2}$.

Excellent solutions were received from *J. W. YOUNG*, *G. B. M. ZERR*, *J. SCHEFFER*, and *W. H. DRANE*.

CALCULUS.

98. Proposed by CHARLES CARROLL CROSS, Meredithville, Va.

Of the circumference of a fixed circle radius R rolls a circle radius r. Required the length of the curve described by a point on the circumference of the rolling circle; (1) when the circle rolls on the inside; (2) when the circle rolls on the outside of the circumference of the fixed circle.

Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; WALTER H. DRANE, A. M., Graduate Student, Harvard University, Cambridge, Mass.; J. SCHEFFER, A. M., Hagerstown, Md.; M. E. GRABER, Student, Heidelberg University, Tiffin, O.; and G. B. M. ZERR, A.M., Ph.D., Professor of Science and Mathematics, Chester High School, Chester, Pa.

We have here the epicycloid and hypocycloid. The equation of the former is

$$x=(R+r)\cos\phi-r\cos\frac{R+r}{r}\phi,\text{ and }y=(R+r)\sin\phi-r\sin\frac{R+r}{r}\phi.$$

$$\therefore\frac{dx}{d\phi}=-(R+r)\sin\phi+(R+r)\sin\frac{R+r}{r}\phi.$$